PHY 571: Quantum Physics

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Background Topics continued

Module 1, Lectures 4 and 5

Useful constants in the Bohr atom

- Fine structure constant: $\alpha = e^2/\hbar c \simeq 1/137$; the value is given by $\alpha^{-1} = 137.0359895(61)$, dimensionless
- $(v/c) = Z\alpha/n$, is the electron velocity relativistic or not? H-atom, no, inner cores of heavy atoms, yes
- Ground state energy $E = -\frac{1}{2}mc^2.(Z\alpha/n)^2$; mc^2 = rest energy of electron = 511 keV. So 100 eV e^- is not relativistic, 1 MeV is (LEED and HVEM energies)
- Radius of lowest Bohr orbit: $a_0 = (\hbar/mc).Z\alpha$; $Z\alpha$ is a number, $(\hbar/mc)$ is the Compton wavelength/2π. The Z=1 value of $a_0 = 5.2977249(24)*10^{-11}$ m, i.e.0.530 Å. This H-atom radius is known as the Bohr radius.

More useful constants and exercises

- Time and Frequency: $(\hbar/mc^2) \simeq 1.3*10^{-21}$ seconds is a characteristic time; thus $(mc^2/\hbar)$ is a frequency.

- Use the various Bohr formulae to show that the $n_2 = 2$ to $n_1 = 1$ transition has a transition energy ΔE and angular frequency ω, given by

  $\Delta E = 13.6(1 - 1/4) = 10.2$ eV and

  $\omega = (\alpha^2/2)(1 - 1/4)(mc^2/\hbar) \simeq 1.5*10^{-16}$ radians/sec

- Now you are in a good position to answer all the short questions on Problem Set #1, and to be completely clear about units, both c.g.s (Gasiorowicz, 2nd Ed) and m.k.s. (3rd Ed). Hand in these easy problems in *asap*.

Topic 6: The Uncertainty Principle

- Wave-particle duality: light and matter can behave as either particles or waves. *How is this possible?* We have to encompass both possibilities, *that's the trick*
- Uncertainty relates to conjugate variables. These are [x, p_x] or [t, E] + some others, in the form $\Delta x.\Delta p_x \geq \hbar$
- Classic interference experiments demonstrate wave-like behavior: Young's two-slit e^- experiment; also Heisenberg's γ-ray microscope and resolution limits
- Photon or particle counters, however, show that both e^- and γ-rays arrive as quanta, click-click, as in the Geiger counter used to detect environmental radiation

Uncertainty Principle: examples and exercises

- Draw both types of "wave" experiments, and identify the conjugate variables involved; think of other examples: Problem set #1 Gedanken (thought) expt
- Visualize a "particle" as a wave-packet, and draw this, and/or explore examples using the web resources
- Think how one may write down the function f(**x**) describing a wave-packet which contains a spread $\Delta \mathbf{x}$ and $\Delta \mathbf{p}$, or equivalently a spread of wave vectors $\Delta \mathbf{k}$
- Explore the general relation between f(x) and g(k) in 1-dimension: these are Fourier Transform pairs. Web pages explore projects and other applications

Topic 7: Fourier Transform Pairs

- Work through the single slit, sometimes call the "top hat" function for $f(x)$. *What is the diffraction amplitude g(k)?*
- Reminders of cos(kx) and sin(kx) in complex number form, expressed in terms of $\exp(\pm ikx)$, $i = \sqrt{-1}$
- Gaussian function $f(x) = \exp(-x^2/2\alpha)$ (Gasiorowicz) or $f(x) = \exp(-(x-x_0)^2/2a^2)$ (Liboff), *what is g(k)?* Quantitative measures of Δx, Δk, hence $\Delta x.\Delta k = \#^*\hbar$
- Lorentzian lineshape $g(\omega) = N/(\omega^2 + a^2)$, *what is f(t)?*, related to shape of spectral lines (N = normalization). See projects http://venables.asu.edu/quant/fourier.html

Two-slit Interference and Convolution

- Draw $f(x)$ = two "top hats" width a, separated by distance b: write as a convolution
- $f(x) = [\delta(x-b/2) + \delta(x+b/2)] * f_{top}(x)$
- convolution = integral $[f_{lattice}(x')].f_{shape}(x-x')dx'$
- $g(k)$ = product $(g_{lattice}(k) \,.\, g_{shape}(k))$, i.e. if

  $f(x) = f_1(x) * f_2(x)$, then $g(k) = g_1(k).g_2(k)$

  g_1 = interference pattern

  g_2 = diffraction envelope

See Fourier handouts (Cowley), web supplement 2A and Student project