Blackbody radiation

BR - radiation emitted by a totally absorbing surface (a black body)

$W(\lambda, T)$ is the radiation energy density

$[W] = \text{erg cm}^{-3}$

Energy flux, $[E] = \frac{\text{erg}}{\text{cm}^2 \cdot \text{s}}$

Box with radiation bouncing off the walls

Problem 1-1:

$$dW = W(\lambda, T) \, dr$$

$$dE = \frac{c \, W}{4 \pi}$$

Wien's Law:

$$W(\lambda, T) = \frac{f(\lambda T)}{\lambda^5}$$

Radiation flux (or $W$) was found to pass through a maximum, $\lambda_{\text{max}}$

$$\lambda_{\text{max}} = \frac{6}{T}$$

Energy density in frequency units:

$$u(\nu, T) = W(\lambda, T) \left| \frac{d\lambda}{d\nu} \right| = W \left( \frac{c}{\nu}, T \right) \frac{c}{\nu^2}$$

$$\nu = \frac{c}{\lambda}$$
Rayleigh prediction (ultraviolet catastrophe):

- each oscillator mode \( \propto kT \) in energy
- number of modes

\[
\lambda \quad \quad \quad N \propto \left( \frac{\nu}{\lambda} \right)^3, \quad \nu = \nu^3
\]

\[
\frac{dN}{vd\nu} \propto \nu^2
\]

Classical density of modes:

\[
U(\nu, T) = \frac{8\pi \nu^2}{c^3} kT
\]

Planck's formula:

Planck's constant

\[
U(\nu, T) = \frac{8\pi G}{c^3} \frac{\nu}{e^{h\nu/kT} - 1}, \quad \beta = \frac{1}{kT}
\]

At low \( \nu \):

\( \nu h < < 1 \), \( e^{h\nu/kT} - 1 \approx \nu h \)

\[
U(\nu, T) = \frac{8\pi \nu^2}{c^3} kT, \textit{ Rayleigh's result}
\]
Total energy density (energy per unit volume):

\[ U(T) = \frac{8\pi^4}{c^3} \int_0^{v^3 dV} \frac{d^3}{c^3 \epsilon^4} \propto T^4 \int_0^\infty dx \frac{x^3}{e^x - 1} \]

\[ U(T) = aT^4 \quad \text{Stefan-Boltzmann law} \]

Main physical result: The energy per degree of freedom \((kT \text{ in classical physics})\) is not constant, but depends on the "excitation frequency".

Planck's idea: Material of the cavity absorbs and emits radiations in quanta of energy \(h\nu\).

Einstein's concept: Radiation itself is composed of quanta of energy, photons.

Example: Radiation falling to the Earth

\[ T = 6000 \, \text{K} \]

\[ \text{Sun} \quad \bigcirc \quad \text{Earth} \]

\[ R_s = 7 \times 10^8 \, \text{m} \]

\[ R = 1.5 \times 10^{11} \, \text{m} \]

\[ E(T) = \frac{c}{4} U(T) \quad \text{per unit area} \]

\[ A = 4\pi R_s^2 \]

\[ E_E = \left( \frac{R_s}{R} \right)^2 \frac{c}{4} U(T) \approx 1.6 \, kW \quad \text{(per } m^2 \text{)} \]
1-2. The photoelectric effect

$PE = \text{emission of electrons from solids by radiation}$

Observations:
- Electrons are emitted
- Wavelength dependence (threshold)
- $i \propto$ Intensity of light
- $E_e$ depends on frequency

\[ ke = \frac{1}{2} m v^2 = h\nu - \Phi \]

... work function

\[ ke = eV_{\text{stop}} \quad \text{potential required to stop the electrons} \]

$V_0 = \frac{\Phi}{h}\nu$

E. 1-4: $\lambda = 290 \text{ nm}$, $W = 4 \text{ eV}$

$-eV_{\text{stop}} = ke = h\nu - W$

\[ h\nu = \frac{34982}{8065} = 4.27 \text{ eV} \quad -eV_{\text{stop}} = 0.27 \text{ eV} \]

\[ \frac{1}{\lambda} = 34882 \text{ cm}^{-1} \]

$8065 \text{ cm}^{-1} = 1 \text{ eV}$
1-2 The Compton effect

Basic observation: two peaks in the scattered X-ray light. The position of the new red-shifted peak depends on the scattering angle.

\[ P \rightarrow \vec{P} \]  
\[ \vec{P}' \rightarrow \text{scattered photon} \]  
\[ \vec{P} \rightarrow \text{recoiling electron} \]  
\[ \vec{P} \rightarrow \text{source} \]

New peak

\[ I \]

Relativistic kinematics:

\[ E = \left[ (m_0 c^2)^2 + (pc)^2 \right]^{1/2} \]

\[ v = c = \frac{\partial E}{\partial P} = \frac{pc}{(m_0^2 c^4 + p^2 c^2)^{1/2}} \]

\[ m_0 = 0 \]

\[ E = pc \rightarrow \text{momentum of the photon} \]

Momentum conservation:

\[ \vec{P} = \vec{P}' + \vec{P} \]

\[ P^2 = P^2 + P'^2 - 2PP' \cos \Theta \]

Energy conservation:

\[ h\nu + mc^2 = h\nu' + \left( mc^2 + p'c^2 \right)^{1/2} \]

\[ mc^2 + P'^2 = (h\Delta \nu + m^2 c^4)^{1/2} \]

\[ \frac{P^2}{c^2} = \left( \frac{h\nu}{c} \right)^2 + \left( \frac{h\nu'}{c} \right)^2 - 2 \frac{h^2 \nu \nu'}{c^2} \cos \Theta \]
\[ p^2 c^2 = h^2 (\nu - \nu')^2 + 2 h^2 \nu \nu' (1 - \cos \theta) \]

\[ (\hbar \nu + mc^2)^2 - mc^2 c^4 = h \nu^2 + 2 h \nu \nu' (1 - \cos \theta) \]

\[ h \nu \nu' (1 - \cos \theta) = mc^2 \Delta \nu \]

\[ \frac{h}{mc^2} (1 - \cos \theta) = \frac{1}{\nu_1} - \frac{1}{\nu} \rightarrow \lambda' - \lambda = \frac{h}{mc^2} (1 - \cos \theta) > 0 \]

\[ \frac{\lambda}{mc} = 2.426 \times 10^{-13} \text{ m} \]

**Compton wavelength**

**Problem 1-5**: \( E(\gamma) = 100 \text{ MeV} \)

\[ q \rightarrow \gamma - q' \]

\[ q \rightarrow q' \]

\[ q c + Mc^2 = 1q' c + \sqrt{(Mc^2)^2 + (q-q')^2 c^2} \]

\[ q > 0 \quad [ (q - q')c + Mc^2 ]^2 = (Mc^2)^2 + (q - q')^2 c^2 \]

\[ q - q' = 0, \quad q' = q \quad \text{photons has elastic scattering} \]

\[ q' < 0 \quad [(q + q')c + Mc^2]^2 = (Mc^2)^2 + (q + q')^2 c^2 \]

\[ 2c^3M (q + q') = (q - q')^2 - (q + q')^2 c^2 = -4q'q c^2 \]

\[ q + q' = -\frac{2q'q}{Mc}, \quad q' = -\frac{q}{1 + \frac{2q}{Mc}} \]

\[ \Delta E = (q + q')c = \frac{2(h\nu)^2}{Mc^2} \frac{1}{1 + \frac{2h\nu}{Mc^2}}, \quad Mc^2 \approx 1 \text{ GeV} \]

\[ \Delta E = \frac{2 \times 0.1^2}{1 + \frac{2 \times 0.1}{1}} = 16 \text{ MeV} \]
de Broglie suggestion: every particle with momentum $p$ can be assigned the wavelength $\lambda = \frac{h}{p}$

Davisson-Germer experiment: electron diffraction by regular atomic arrays forming the crystal

\[ 2\lambda \sin \theta = n\lambda \]

interference of two scattered beams of electrons for a maximum

1-5 The Bohr atom

Rutherford planetary model: electrons travel around the heavy nucleus located at the atom's center

Problem: according to classical electrodynamics electrons should lose their energy in about $10^{-10}$ s and fall on the nucleus

Experiment: discrete spectral lines of H:

\[ \frac{1}{\lambda} = \text{Ry} \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad (\text{Balmer formula}) \]

Ry is the Rydberg constant
The Bohr postulates:

1. An atomic system exists in a discrete set of stationary states.
2. Absorption and emission correspond to transitions between two stationary states.
3. Radiation \( \nu = E_i - E_f \) \((E_i > E_f)\).
4. Allowed orbits are determined by quantized angular momentum:
   \[ mvr = \hbar n, \quad \hbar = \frac{h}{2\pi} \quad (\text{k} \text{e} \text{a} \text{r}) \]

   Equilibrium condition:
   \[ \frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{mev^2}{r} \]
   \[ mvr = n\hbar \]

   \[ E_n = -13.6 \text{eV} \frac{Z^2}{n^2} \]
   \[ r_n = n^2 \frac{a_0}{Z}, \quad a_0 = 0.529 \text{ Å} \]

   Bohr radius

5. Fine structure constant:
   \[ \alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137} \]

6. Orbital velocity:
   \[ v_n = \frac{Ze}{n^2} \]

7. More general quantization rule:
   \[ \int p \, dx = 2\pi\hbar n \]
   \( \text{Integral over the orbit} \)
Stern-Gerlach experiment (1922):

Ag atoms possessing a magnetic moment $\mu$

\[ V = -\mu \cdot B \]

\[ F = -\nabla V = D(\mu \cdot B) \approx M_0 \frac{d^2 B^2}{d z^2} \]

The classical expectation is that the magnetic moments are distributed between $+\mu$ and $-\mu$. One would expect:

\[ \theta = 0 \]

continuous band of adsorbed silver

One finds instead:

\[ \theta = 0 \]

The experiment suggested that the angular momentum cannot take all possible values but instead is quantized. Qualitatively, it showed that the Bohr hypothesis was correct, but quantitatively it cannot stand against the experimental tests.
The correspondence principle

Classical limit is reached at large quantum numbers

Radiation of photons by Bohr atom:

$$ frequency \ \nu_{ce} = \frac{\nu}{2\pi r} \propto \frac{1}{n^3} $$

$$ quantum \ frequency: \ \nu = 13.6 \text{eV} \ \frac{Z^2}{n^2} \left( \frac{1}{n^2} - \frac{1}{(n+1)^2} \right) \approx \frac{27.2 \text{eV} \ \frac{Z^2}{n^3}}{n \gg 1} $$

$$ \nu \rightarrow \nu_{ce} \quad n \gg 1 $$