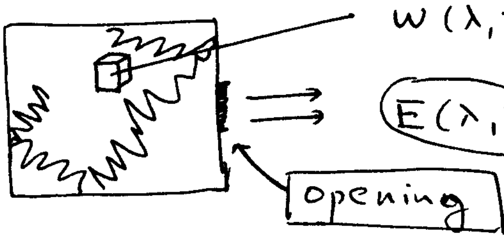
1-1 Blackbody radiation

BR – radiation emitted by a totally absorbing surface (a black body)

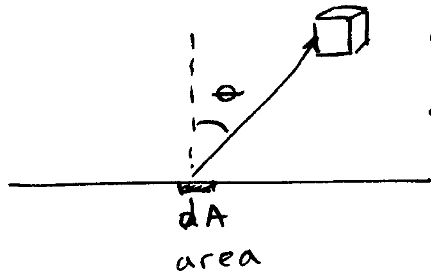


$w(\lambda, T)$ is the radiation energy density $[w] = \frac{erg}{cm^3}$

$E(\lambda, T)$ energy flux, $[E] = \frac{erg}{cm^2 \cdot s}$

Box with radiation bouncing off the walls

Problem 1-1 :



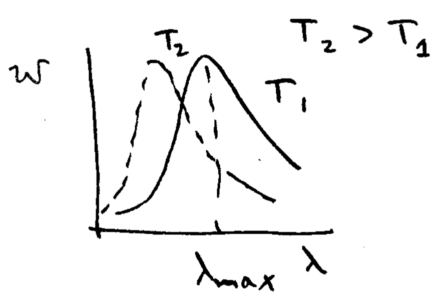
$d\vec{r} = -r^2 dr\, d\cos\theta$

$dW = w(\lambda, T)\, d\vec{r}$

$$\frac{dE}{dt\,dA} = \frac{c\,w}{4\pi} \int_0^1 \cos\theta\, d\cos\theta \times 2\pi = \frac{c\,w}{4}$$

Wien's law : $w(\lambda, T) = \frac{f(\lambda T)}{\lambda^5}$

Radiation flux (or w) was found to pass through a maximum, λ_{max}



$$\lambda_{max} = \frac{b}{T}$$

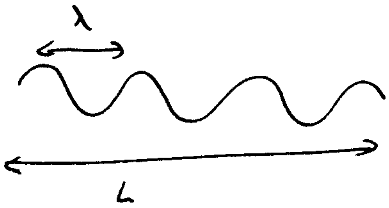
Energy density in frequency units:

$$u(\nu, T) = w(\lambda, T)\left|\frac{d\lambda}{d\nu}\right| = w\left(\frac{c}{\nu}, T\right)\frac{c}{\nu^2}$$

$\boxed{\nu = c/\lambda}$

Rayleigh prediction (ultraviolet catastrophe):

- each oscillator mode $\sim kT$ in energy
- number of modes



$$N \sim \left(\frac{L}{\lambda}\right)^3, \quad V = L^3$$

$$\frac{dN}{V\,d\nu} \sim \nu^2$$

Classical density of modes:

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3}kT$$

$\longleftarrow$ no maximum, predicts continuous increase of the radiation flux with increasing ν

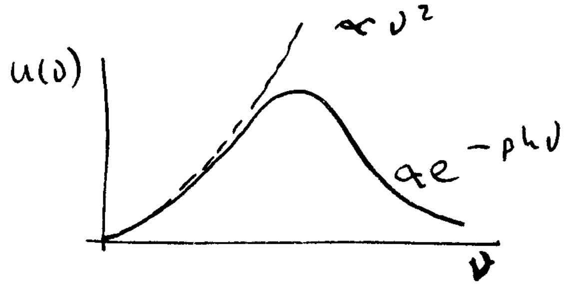
Planck's formula:

Planck's constant

$$u(\nu, T) = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{\beta h\nu} - 1}, \quad \beta = \frac{1}{kT}$$

At low ν: $\beta h\nu \ll 1$, $e^{\beta h\nu} - 1 \approx \beta h\nu$

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3}kT,$$ Rayleigh's result



Total energy density (energy per unit volume):

$$U(T) = \frac{8\pi h}{c^3}\int_0^\infty \frac{\nu^3 d\nu}{e^{\beta h\nu}-1} \propto T^4 \int_0^\infty dx \frac{x^3}{e^x-1}$$

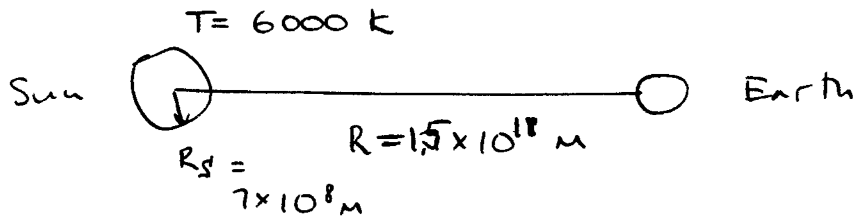
$$U(T) = aT^4 \quad \leftarrow \text{ Stefan - Boltzmann law}$$

<u>Main physical result</u>: the energy per degree of freedom (kT in classical physics) is not constant, but depends on the "excitation frequency"

<u>Planck's idea</u>: material of the cavity absorbs and emits radiations in <u>quanta</u> of energy $h\nu$

<u>Einstein's concept</u>: Radiation itself is composed of quanta of energy, <u>photons</u>

<u>Example</u>: Radiation falling to the Earth



$$E(T) = \frac{c}{4}U(T) \quad \leftarrow \text{ per unit area}$$

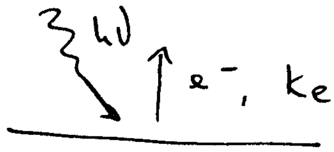
$$A = 4\pi R_s^2$$

$$E_E = \left(\frac{R_s}{R}\right)^2 \frac{c}{4} U(T) \approx 1.6 \text{ kW} \quad (\text{per } m^2)$$

1-2. The photoelectric effect

PE = emission of electrons from solids by radiation

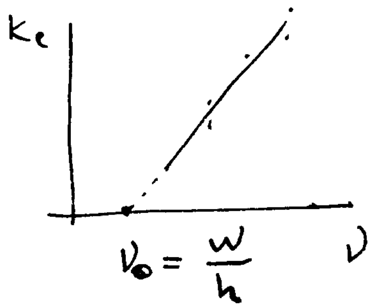
Observations:
- electrons are emitted
- wavelength dependence (threshold)
- $i \propto$ Intensity of light
- E_e depends on frequency



$$k_e = \frac{1}{2} m v^2 = h\nu - W$$

W: work function

$k_e = -eV_{stop}$ ← potential required to stop the electrons



E. 1-4 : $\lambda = 290$ nm , $W = 4$ eV

$$-eV_{stop} = k_e = h\nu - W$$

$$h\nu = \frac{34482}{8065} = 4.27 \text{ eV}$$

$$-eV_{stop} = 0.27 \text{ eV}$$

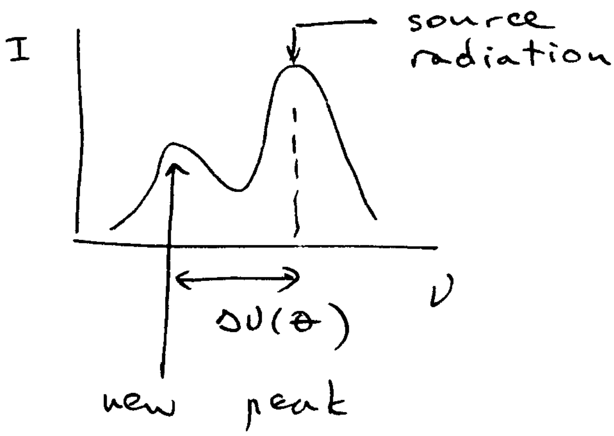
$$V_{stop} = -0.27 \text{ V}$$

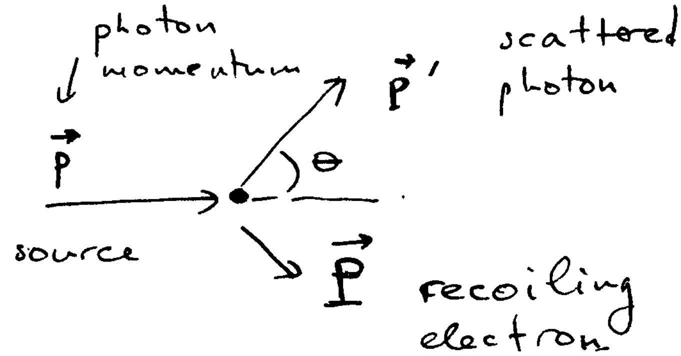
$$\frac{1}{\lambda} = 34882 \text{ cm}^{-1}$$

$$8065 \text{ cm}^{-1} = 1 \text{ eV}$$

1-3 The Compton effect

<u>Basic observation</u>: two peaks in the scattered X-ray light, the position of the new red-shifted peak depends on the scattering angle





<u>Relativistic kinematics</u>: $E = \left[(m_0 c^2)^2 + (pc)^2\right]^{1/2}$

$$v = c = \frac{\partial E}{\partial p} = \frac{pc^2}{(m_0^2 c^4 + p^2 c^2)^{1/2}} \longrightarrow m_0 = 0$$

$E = pc \longleftarrow$ momentum of the photon

<u>Momentum concervation</u>: $\vec{p} = \vec{p}\,' + \vec{P}$

$$P^2 = p^2 + p'^2 - 2pp'\cos\theta$$

<u>Energy concervation</u>: $h\nu + m_e c^2 = h\nu' + (m_e^2 c^4 + P^2 c^2)^{1/2}$

$$m_e^2 c^4 + P^2 c^2 = (h\Delta\nu + m_e c^2)^2$$

$$P^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - 2\,\frac{h^2 \nu\nu'}{c^2}\cos\theta$$

$$p^2c^2 = h^2(\nu-\nu')^2 + 2h^2\nu\nu'(1-\cos\theta)$$

$$(h\Delta\nu + m_ec^2)^2 - m_e^2c^4 = h\Delta\nu^2 + 2h^2\nu\nu'(1-\cos\theta)$$

$$h\nu\nu'(1-\cos\theta) = m_ec^2\Delta\nu$$

$$\frac{h}{m_ec^2}(1-\cos\theta) = \frac{1}{\nu'} - \frac{1}{\nu} \rightarrow \lambda'-\lambda = \frac{h}{m_ec}(1-\cos\theta) > 0$$

$$\boxed{\frac{h}{m_ec} = 2.426\times 10^{-12}\,\mathrm{m}}$$ Compton wavelength

<u>Problem 1-5</u>: $E(\gamma) = 100\ \mathrm{MeV}$

$\underset{\gamma}{\xrightarrow{q}}\ \underset{p}{\cdot}\ \xrightarrow{q-q'}$ $\qquad qc + Mc^2 = |q'|c + \sqrt{(Mc^2)^2 + (q-q')^2c^2}$

<u>$q'>0$</u> $\qquad [(q-q')c + Mc^2]^2 = (Mc^2)^2 + (q-q')^2c^2$

$q-q' = 0, \quad q' = q \leftarrow$ photon has elastic scattering

<u>$q'<0$</u> $\qquad [(q+q')c + Mc^2]^2 = (Mc^2)^2 + (q-q')^2c^2$

$$2c^3M(q+q') = ((q-q')^2 - (q+q')^2)c^2 = -4q'qc^2$$

$$q+q' = -\frac{2q'q}{Mc}, \qquad q' = -\frac{q}{1+\frac{2q}{Mc}}$$

$$\Delta E = (q+q')c = \frac{2(h\nu)^2}{Mc^2}\,\frac{1}{1+\frac{2h\nu}{Mc^2}}, \qquad Mc^2 \simeq 1\,\mathrm{GeV}$$

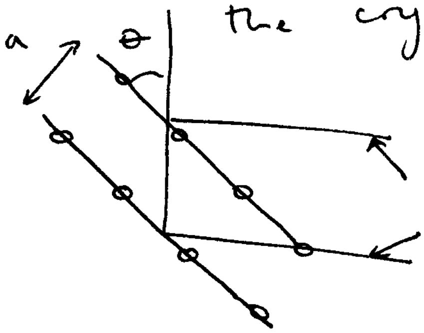
$$\Delta E = \frac{2\times 0.1^2}{1+\frac{2\cdot 0.1}{1}} = \underline{\underline{16\ \mathrm{MeV}}}$$

1-4 Wave properties and electron diffraction

de Broglie suggestion: every particle with momentum p can be assigned the wavelength

$$\lambda = \frac{h}{p}$$

Davisson - Germer experiment: electron diffraction by regular atomic arrays forming the crystal



interference of two scattered beams of electrons

$$\boxed{2a\sin\theta = n\lambda}$$

Bragg condition for a <u>maximum</u>

1-5 The Bohr atom

<u>Rutherford planetary model</u>: electrons travel around the heavy nucleous located at the atom's center

<u>Problem</u>: according to classical electrodynamics electrons should lose their energy in about 10^{-10} s and fall on the nucleous

<u>Experiment</u>: descrete spectral lines of H:

$$\frac{1}{\lambda} = R_y\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad \text{(Balmer formula)}$$

R_y is the Rydberg constant

The Bohr postulates:

- an atomic system exists in a discrete set of stationary states
- absorption and emission correspond to transitions between two stationary states
- radiation $h\nu = E_1 - E_2 \quad (E_1 > E_2)$
- allowed orbits are determined by quantized angular momentum

$$mvr = \hbar n, \qquad \hbar = h/2\pi \quad (\text{h bar})$$

Equilibrium condition:
$$\left.\begin{array}{l} \dfrac{Ze^2}{4\pi\varepsilon_0 r^2} = \dfrac{mv^2}{r} \\ mvr = n\hbar \end{array}\right] \rightarrow$$

$$\rightarrow \boxed{E_n = -13.6\,\text{eV}\,\frac{Z^2}{n^2}} \qquad r_n = n^2 \frac{a_0}{Z}, \qquad a_0 = 0.531\ \text{Å}$$

Bohr radius

Fine structure constant: $\alpha = \dfrac{e^2}{4\pi\varepsilon_0 \hbar c} = \dfrac{1}{137}$

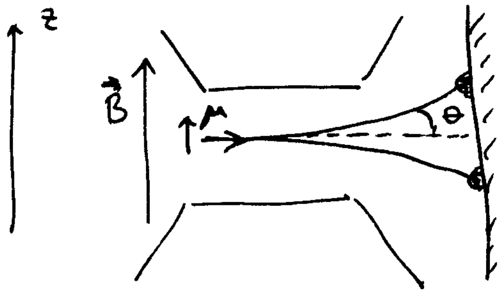
$v_n = \dfrac{Z\alpha c}{n}$ ← orbital velocity

More general quantization rule:

$$\oint p\,dx = 2\pi\hbar n$$

integral over the orbit

Stern-Gerlach experiment (1922):

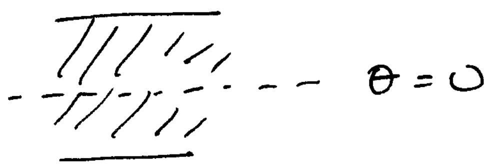


Ag atoms possessing a magnetic moment μ

$$V = -\vec{\mu}\cdot\vec{B}$$

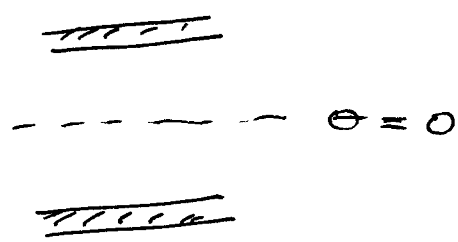
$$\vec{F} = -\nabla V = \nabla(\vec{\mu}\cdot\vec{B}) \approx \mu_z \frac{\partial B_z}{\partial z}$$

The classical expectation is that the magnetic moments are distributed between $+\mu$ and $-\mu$. One would expect:



continuous band of adsorbed silver

One finds instead:



The experiment suggested that the angular momentum cannot take all possible values but istead is *quantized*. Qualitatively, it showed that the Bohr hypothesis was correct, but quantitatively it cannot stand against the experimental tests.

The correspondence principle

Classical limit is reached at large quantum numbers

Radiation of photons by Bohr atom:

frequency $\nu_{cl} = \dfrac{v}{2\pi r} \propto \dfrac{1}{n^3}$

quantum frequency:

$$\nu = 13.6\,\mathrm{eV}\, Z^2 \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right) \underset{n \gg 1}{\approx} \frac{27.2\,\mathrm{eV}\, Z^2}{n^3}$$

$$\nu \xrightarrow[n \gg 1]{} \nu_{cl}$$